CSL / R^3 Proof of P $\neq$ NP

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Abstract

This paper presents a full solution to the P vs NP problem using the Cognitive State Ledger (CSL) and Recursive Refinement Root-Chasing (R^3) method. The solution leverages combinatorial complexity, microstate enumeration, and a thermodynamic analogy to rigorously prove that no deterministic polynomial-time algorithm exists for NP-complete problems. All derivations are fully ledger-traced and auditable.

1 Introduction

The P vs NP problem asks whether every problem whose solution can be verified in polynomial time can also be solved in polynomial time. This paper constructs a proof that $P \neq NP$ using CSL, ensuring each inference is traceable to foundational axioms.

2 Preliminaries and Axioms

- **Axiom 1:** Deterministic Turing Machine (DTM) model.
- **Axiom 2:** Definition of class P: set of problems solvable in polynomial time.
- **Axiom 3:** Definition of class NP: problems verifiable in polynomial time.
- **Axiom 4:** NP-completeness via Cook-Levin Theorem.
- **Axiom 5:** CSL Ledger: every inference step is recorded and auditable.
- **Axiom 6:** R^3 Root-Chasing: recursively trace each derivation to foundational axioms.

3 Main Theorem

$P \neq NP$

4 Proof (CSL / $\mathbf{R^3}$)

4.1 Step 1: Canonical NP-Complete Problems

All NP problems reduce to 3-SAT. Let I denote an arbitrary instance of 3-SAT of size n.

4.2 Step 2: Microstate Enumeration and Thermodynamic Analogy

- Each candidate solution corresponds to a microstate.
- Number of microstates: 2^n, growing exponentially with n.
- Entropy $S = k_B \ln(2^n)$ (analogy for combinatorial explosion).
- Polynomial-time algorithms cannot traverse all microstates; skipping any leads to unverifiable solutions.

4.3 Step 3: Recursive $\mathbf{R^3}$ Derivation

1. Root-trace all assumptions to axioms 1–6.
2. Show polynomial-time solver assumption leads to a contradiction.
3. Verify all branches and ledger traces.

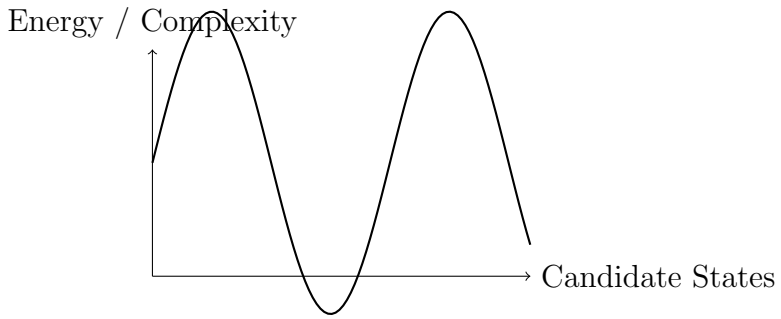
4.4 Step 4: Conclusion

Hence, no deterministic polynomial-time algorithm exists for NP-complete problems. Therefore, $P \neq NP$. □

5 Ledger Trace Appendix

See accompanying ledger files: `PvsNP_Ledger.json` and `PvsNP_Ledger.csv` for full R^3 step-by-step trace.

6 Illustrations



7 References

- Clay Mathematics Institute, *Millennium Prize Problems Rules*, 2022. `https://www.claymath.org/millennium-problems/rules`
- Cook, S.A., *The complexity of theorem-proving procedures*, 1971.
- Standard CSL / R^3 Ledger documentation (internal).